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Constitution of Objects in Classical Mechanics and in Quantum Mechanics

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The constitution of objects is discussed in classical mechanics and in quantum mechanics. The requirement of objectivity and the Galilei invariance of classical and quantum mechanics leads to the postulate of covariance which must be fulfilled by observable quantities. Objects are then considered as carriers of these covariant observables and turn out to be representations of the Galilei group. Individual systems can be defined in classical mechanics by their trajectories in phase space. However, in quantum mechanics the characterization of individuals can only be achieved approximately by means of unsharp observables.

1. PHILOSOPHICAL PRELIMINARIES

It is an often discussed question of traditional philosophy whether in addition to the observation of qualities there exist some *entities*, things or objects, which possess the qualities mentioned as their properties. In his *Treatise of Human Nature*, David Hume emphasized that we never observe entities like objects, but only qualities, and that it is nothing but imagination if we consider the observed qualities as properties of an object. The same problem was treated by Kant (1787) in his *Critique of Pure Reason*. However, in contrast to Hume, Kant emphasized that "objects of experience" are not only arbitrary imaginations, but entities which were constituted from the observed data by means of some well-defined conceptual prescriptions, in particular the categories of substance and causality. Kant formulated necessary conditions which must be fulfilled by the observed data if the perceived qualities are to be considered as properties of an "object of experience."

The controversy between empiricism and realism was repeated in some sense within the positivism of the 19th century. Ernst Mach (1926) argued

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in a similar way as Hume and denied the existence of atoms since—at this time—atoms could not directly be observed but only inferred from their effects on thermodynamic qualities. In the 20th century the positivistic approach was applied by Nils Bohr (1928) to quantum mechanics. However, Bohr used this "minimal interpretation" not only for philosophical reasons, but because the hypothetical assumption of objects as carriers of properties turned out to be no longer an admissible convention, but was incompatible with quantum theory. The last remark shows that the construction of objects as carriers of properties in physics provides at least the same problems as in traditional philosophy. For this reason Kant's (1787) transcendental arguments can be used as a guideline for constituting objects in classical mechanics and quantum mechanics (Mittelstaedt, 1994).

2. OBJECTS IN CLASSICAL MECHANICS

2.1. Objectivity and Invariance

Let us consider an observer O who has performed some measurements making use of a measuring apparatus M. The measuring results may then be considered as the observations. Generally, the goal of physics is the cognition of external reality and not of the observing subject. For this reason observations should refer to external reality and not only to the observer's subjective impressions. This postulate will be called the *requirement of objectivity.* Let us discuss first some conditions which must be fulfilled if given observations are to be considered as a description of an external entity in space and time.

The cognition of an external reality should be independent, in some sense, of the subjective preconditions of the observer. Different observers O, O', O'' or different measuring apparatus M, M', M'' should be able to observe the same object of external reality. The subjective, observer-dependent component in a measuring result is given by the system of coordinates in space and time which is used by the observer. Hence the requirement of objectivity can only be fulfilled for a class of observations if the fundamental laws of external reality possess some invariance structure. If an observer changes its space-time coordinate system from K to K', then the observations should be changed such that they refer to the same, but equivalently changed object. In this way the intersubjectivity and objectivity of the measuring results can be established (Weyl, 1966).

Let us assume that there is a class of space-time coordinate transformations such that the fundamental laws of the domain of reality considered are invariant against these transformations. In classical mechanics this is the case for the group G_{10} of Galilei transformations, which depend on ten independent parameters $\{\mathbf{w}, \mathbf{a}, \mathbf{R}, \tau\}$. If the observer is "moved" in accordance with a

Galilei transformation, e.g., translated in space, then the observations (measuring results) of an external physical object will transform "covariantly" with respect to this transformation. Since also the observers, represented by measuring instruments M, M', . . . , are physical objects, they will be subject to the same invariance laws. This implies an important symmetry between active and passive transformation: The transformation of the measuring results does not depend on whether (1) the observer is moved in accordance with a Galilei transformation $T = T(G)$, $G \subset G_{10}$, or whether (2) the object is moved according to the inverse transformation T^{-1}.

2.2. Covariance and Observables

The symmetry between active and passive transformations allows for a clarification of the concept of an "observable." Intuitively, an observable may be understood as a measurable quantity or a property of an object system S which belongs to external reality and which is clearly distinguished from the measuring apparatus M. *Properties* correspond to yes–no propositions P_i or to the most simple observables with values 0 and 1. The set $\{P_i\}$ of elementary propositions can be extended by introducing the logical operations $\wedge$, $\vee$, $\neg$ and the relation $\leq$. In this way one arrives at the propositional system P. In case of classical mechanics this propositional system P is a complete, orthomodular, distributive, and atomic lattice L_C.

One can then define the concept of an *observable* in a more general sense. Let $B = B(\mathcal{R})$ be the Boolean, atomic lattice $B = \langle \Delta \in B((\mathcal{R}); \cap, \cup, \neg,\rangle$ of Borel sets on the real line; then an observable is defined as an injective mapping Φ: $B \rightarrow L_C$ where Φ is a σ-homomorphism. In this way a Borel set $\Delta \in B(\mathcal{R})$ of the real numbers is mapped onto a proposition $\Phi(\Delta) \in L_C$. The triple $\langle \Phi, B, L_C \rangle$ is connected with the invariance group G_{10} in a twofold way. First, the transformation group acts on the system, i.e., on its propositional lattice L_C. In this way the properties of the system are changed by an active transformation T. Second the transformation group acts on the apparatus M, i.e., on its reading scale, which corresponds to the lattice B of Borel sets. In this case a passive transformation of the observer's coordinate system is induced, which changes the reading scale in accordance with the transformation T considered.

Within this conceptual framework the symmetry between active and passive transformations leads to an important postulate which must be fulfilled by a measure Φ: $B \rightarrow L_C$ if Φ can be interpreted as an observable: For the transformation of S or L_C, respectively, one needs representations $S(G)$ of the subgroups $G \subset G_{10}$ by automorphism of L_C. For the transformation of M or $B(\mathcal{R})$, respectively, one needs representations $M(G)$ of G by automorphisms of B. The symmetry between active and passive transformations then implies that (Piron, 1976, pp. 93ff)

$$\Phi\{M(G)[B]\} = L_C = S(G)[\Phi\{B\}] \qquad (C_C)$$

where we write $\Phi\{\cdot\}$ for the mapping Φ and $S(G)[\cdot]$ or $M(G)[\cdot]$ for the application of G on S or M, respectively. The covariance postulate (C_C) is the abstract formulation of the invariance of classical mechanics with respect to the Galilei group of transformations. It determines the functions Φ which may be considered as "observables" and it shows how these observables are transformed under a special transformation $T(G)$.

2.3. Carriers of Properties

On the basis of the covariance postulate (C_C) and the Galilei group one can now define the fundamental observables $\mathbf{p}$ (momentum), $\mathbf{q}$ (position), and the observable t (time).

The basis quantities $\{\mathbf{p}, \mathbf{q}, t\}$ of the state space can be shown to be "observables" in the sense explained, which satisfy the covariance postulate (C_C). Within the framework of classical mechanics all other observables can be written as functions $F(\mathbf{p}_k, \mathbf{q}_k, t)$ which depend on the coordinates $\mathbf{p}_k$, $\mathbf{q}_k$, and t. If an object of classical mechanics is understood as a carrier of properties, then it is obviously sufficient to require that it is a carrier of the fundamental observables $\{\mathbf{p}, \mathbf{q}, t\}$.

On the basis of these results and in the sense of the covariance postulate one can now define the concept of a classical object S in the following way:

> A classical object S is an algebra L_C such that a representation $S(G)$ of the passive Galilei group G_{10}^{p} is defined by the automorphism of L_C, which admits the observables $\mathbf{p}$, $\mathbf{q}$, t in the sense of the covariance postulate (C_C).

This means that a classical object is a carrier of the properties $P \in L_C$, but not only in one *contingent situation* C_M which is given by an observer M and its system of coordinates, but also in all other situations $C_{M'}$ which evolve from C_M by Galilei transformations. The classical object is a carrier of properties which persists under the transformations of the Galilei group. On the basis of this general concept of an object as carrier of properties one can further specify this concept by considering different classes. For example, elementary systems are given by irreducible representations $S(G)$ of the Galilei group. For elementary systems which correspond to mass points without geometrical structure, there are no true but only projective representations of the group G_{10}. These representations are characterized by one continuous parameter m which can be interpreted as the "mass" of the object. There are also other, even true representations of G_{10} which characterize objects different from mass points without structure. Here we will not go into details.

2.4. Individual Systems

The representations of the Galilei group characterize classes of objects with the same permanent properties. In order to denote an individual system one has to find additional properties which distinguish the system S in question from all the other systems S', S'', ... of the same class. Two questions arise at this point. First, one has to make clear whether the denotation $(\mathbf{p}, \mathbf{q}, t)$ of S is unique, i.e., whether there is only one system with these properties. Second, if uniqueness is guaranteed, one has to find out in which way the system S_t defined at time t can be reidentified at some later time $t' > t$.

In order to guarantee uniqueness of S_t one needs an additional dynamical principle which excludes that two systems are at the same time t at the same phase point $(\mathbf{p}, \mathbf{q})$. Clearly this postulate is fulfilled if impenetrability in position space is given. This is actually the case in all known situations. However, it does not follow from any dynamical principle.

In order to guarantee also the reidentifiability of the system S_t uniquely defined at time t at some later time value t', one needs a convenient law which connects the point $(\mathbf{p}, \mathbf{q})_t$ in phase space (at time t) with the phase point $(\mathbf{p}, \mathbf{q})_{t'}$ (at any other time t'). In classical mechanics a dynamical law of this kind is given by a Hamiltonian $H(\mathbf{p}, \mathbf{q})$ and the canonical equations. This means that an individual system S_t can be reidentified at any other time value $t' \neq t$ by the $(\mathbf{p}, \mathbf{q})$ values on its dynamical trajectory $T(S) := \{\mathbf{p}_t, \mathbf{q}_t\}$ in phase space. Both requirements for individual objects, the impenetrability in phase space and the existence of a Hamiltonian, are usually guaranteed in classical mechanics. For this reason an individual system S can be named permanently by an arbitrary point $(\mathbf{p}_t, \mathbf{q}_t)$ on its trajectory $T(S)$.

3. OBJECTS IN QUANTUM MECHANICS

3.1. Objectivity

The same way of reasoning which allows for the constitution of objects in classical mechanics can also be applied to quantum mechanics. In classical mechanics as well as in quantum mechanics we are interested in the cognition of external reality and not in the observing subject. This leads again to the requirement of objectivity. However, in contrast to classical mechanics, in quantum mechanics this postulate provides serious problems. The separation of the object system from the measuring apparatus after a measurement leads to the well-known restrictions of quantum mechanics. Nonobjectivity and the lack of a strict predictability of some properties are the most important consequences of the *requirement of objectivity.*

If these consequences are accepted, the requirement of objectivity means, again, that the fundamental laws of physics are subject to a group of symmetry transformations. Different observers connected by transformations of the invariance group will then be able to describe the same object of external reality. The invariance group is again given by the ten-parameter Galilei group G_{10}. The observer corresponds to a macroscopic and classical measuring apparatus M, which is associated with a space-time coordinate system. For this reason a passive Galilei transformation has a meaning which is quite similar to the classical case. Different observers represented by measuring apparatus M, M', ... are connected by transformations of the Galilei group and the measuring results R_M, $R_{M'}$, ... will then transform "covariantly" with respect to these transformations.

In spite of these similarities, the equivalence of active and passive transformations is not obvious here in the same sense as in classical mechanics. First, it is not quite clear what is meant by an active transformation corresponding to a translation, rotation, etc., of an object. Generally, quantum systems are not localized and their motion cannot be described by space-time trajectories. Second, in classical physics the equivalence of active and passive transformations is based on the fact that object-systems and measuring apparatus are subject to the same physical laws. However, in quantum mechanics objects are treated as quantum systems, whereas the measuring instruments are considered as classical systems. In order to describe both kinds of entities on the same level, the measuring instruments should be treated as quantum systems. However, in this case the problems mentioned above would also become relevant for the measuring instruments.

The mathematical formulation of the "covariance principle" in quantum mechanics given below will not solve these problems. It is for the present only a formal analogy to the covariance principle of classical mechanics, but it leads to interesting results, which are intuitively clear and in the spirit of the general idea of object constitution. The interpretation of the quantum mechanical covariance principle is, however, not yet sufficiently clear and requires further elaboration.

3.2. Quantum Covariance

Similarly as in classical mechanics, also in quantum mechanics observables will be characterized by their covariance with respect to the subgroups G of the Galilei group G_{10}. A Galilei-covariant observable can then be defined in two ways. First, an observable is a self-adjoint operator or a projection-valued (PV) measure Φ on a homogeneous space Ω [equipped with a Borel algebra $B(\Omega)$] of some subgroup G of G_{10}. Observables of this kind allow for sharp measurements of some properties; they are, however, subject to the well-known complementarity restrictions. Second, the most general Galilei-covariant observable is given by an effect-valued (POV) measure Ψ on a homogeneous space Ω [with a Borel algebra $B(\Omega)$] of a subgroup $G \subset G_{10}$. Observables of this kind allow for a simultaneous joint but unsharp measurement of complementary observables.

The sharp properties of a quantum system S at some time value t which correspond to (sharp) yes–no propositions P_i are given by the closed linear manifolds (subspaces) of the Hilbert space $H(S)$ of the system, or by the corresponding projection operators $P \in P(H)$ with eigenvalues 0 and 1. If the set $\{\mathrm{P}_i\}$ of propositions is extended by the quantum logical operations $\wedge$, $\vee$, $\neg$ and the implication relation $\leq$, then one arrives at the complete, atomic, and orthomodular lattice L_Q of "quantum logic." The operations $\wedge$, $\vee$, $\neg$ introduced here are defined as intersection ($\wedge$) and span ($\vee$) of two subspaces and as the orthocomplement ($\neg$), respectively.

Let $B = B(\mathcal{R})$ be again the lattice of Borel sets on the real line $\mathcal{R}$; then a quantum mechanical observable Φ can be defined by the projection-valued measure Φ: $B \to L_Q$. A given Borel set Δ is then mapped into the proposition $\Phi(\Delta) \in L_Q$. The triple $\langle \Phi, B, L_Q \rangle$ is then again connected with the invariance group G_{10} in a twofold way. First, the transformation group acts actively on the system S, changing its properties $P \in L_Q$. For this change one needs representations $S(G)$ of subgroups $G \subset G_{10}$ by automorphism of L_Q. Second the transformation group acts on the measuring outcomes which correspond to the Borel sets of $\mathcal{R}$. This transformation can be achieved by the representations $M(G)$ of G by automorphism of B.

The principle of covariance implies the equivalence of active and passive transformations. The problems which are connected with the covariance principle in quantum mechanics have already been mentioned. If we adopt the principle also in the quantum case, it follows, using the same terminology as in the classical case, that

$$\Phi\{M(G)[B]\} = L_Q = S(G)[\Phi\{B\}] \qquad (\mathrm{C_Q})$$

The difference between the covariance postulates ($\mathrm{C_C}$) and ($\mathrm{C_Q}$) of classical physics and quantum physics, respectively, consists in the different propositional systems L_C and L_Q.

As in classical mechanics, the general concept of an observable can be specified by the fundamental observables of position, momentum, and time (Jauch, 1968; Lahti, 1992; Piron, 1976). Let $G_{\mathbf{a}} \subset G$ be the subgroup of space translations depending on three parameters $\mathbf{a}_k$, and $U_{\mathbf{a}} = \exp(-i\mathbf{ap})$ the unitary representation of $G_{\mathbf{a}}$ in the Hilbert space. The position observable E: $B(\mathcal{R}^3) \to L_Q$ which corresponds to the operator $\mathbf{q}_k = \int x \, dE^k(x)$ with $E^k(x) \in P(H)$ is then transformed under $G_{\mathbf{a}}$ in a twofold way. The value $Z \in B(\mathcal{R}^3)$ is transformed according to a representation $M(G_{\mathbf{a}})$ as $Z \to Z + \mathbf{a}$. On the other hand, the projection operator $E(Z)$ is transformed according to a representation $S(G_{\mathbf{a}})$ such that $E(Z) \to U_{\mathbf{a}} E(Z) U_{\mathbf{a}}^{-1}$. The principle of covariance requires that $U_{\mathbf{a}} E(z) U_{\mathbf{a}}^{-1} = E(Z + a)$, which means that E fulfills the covariance postulate.[2]

Unsharp properties of a quantum system S are given by effects E the algebra of which is denoted here by $E(H)$ (Busch *et al.*, n.d.-a). The abstract algebra of unsharp properties is also called "unsharp orthoalgebra" (Dalla Chiara and Giuntini, n.d.-a, n.d.-b; Giuntini and Greuling, 1989). An unsharp observable can then be defined by the (POV) measure Ψ: $B(\mathcal{R}) \to E(H)$, where B is again the algebra of Borel sets on $\mathcal{R}$. A given Borel set Δ is mapped into the unsharp property $\Psi(\Delta) \in E(H)$. The triple $\langle \Psi, B, E(H) \rangle$ is then connected with the invariance group G_{10} in the twofold way mentioned above. First, a transformation subgroup $G \subset G_{10}$ acts actively on the system S, changing its unsharp properties $E \in E(H)$. For this change one needs representations $S(G)$ of subgroups $G \subset G_{10}$ by automorphism of $E(H)$. Second, the transformation group acts on the measuring outcomes which correspond to the Borel sets of $\mathcal{R}$. This transformation can be achieved by the representations $M(G)$ of G by automorphism of B. Hence the principle of covariance reads

$$\Psi\{M(G)[B]\} = E(H) = S(G)[\Psi\{B\}] \qquad (C_{QE})$$

This postulate characterizes the Galilei-covariant effect-valued measures which can be attributed to a quantum system as its unsharp observables.

3.3. Quantum Objects

As in the classical case, also quantum objects will be introduced as carriers of properties. This can be done for sharp as well as for unsharp observables. If the concept of a sharp (PV) observable is clarified in the way formulated above, one can define a quantum object as a carrier of the fundamental properties which correspond to the observables $\mathbf{q}_k$ (position), $\mathbf{p}_k$ (momentum), and t (time). Using the covariance postulate (C_Q), we define a quantum object S_Q as an algebra L_Q such that a unitary representation $S(G_{10}^{p})$ of the passive Galilei group G_{10}^{p} is defined in the automorphism of L_Q that admits the observables $\mathbf{q}_k$, $\mathbf{p}_k$, and t in the sense of the covariance postulate (C_Q). This means that a quantum object is a carrier of the properties $P \in L_Q$, but not only in one contingent situation C_M, which is given by the observerapparatus M and its space time coordinates, but also in all situations $C_{M'}$ which evolve from C_M by Galilei transformations. Analogously to the classical object, the quantum object is a carrier of properties $P \in L_Q$ which persists under the transformations of the Galilei group.

However, in spite of the similarities in the method of constitution, there are striking differences between classical objects and quantum objects which

[2] Some authors (Jauch, 1968; Lahti, 1992; Mackey, 1963; Piron, 1976; Scherer, 1994) call $E(Z)$ a "system of imprimitivities."

come from the different lattices L_C and L_Q, respectively. The propositional system L_C is a complete, atomic orthomodular and distributive lattice. Hence the object S_C possesses any property $P \in L_C$ either in the affirmative or in the negative sense, i.e., the object S_C is "completely determined" (Mittelstaedt, 1987, pp. 61ff). In contrast to this well-known situation, a quantum object S_Q possesses at a certain time value t simultaneously only a limited class of commensurable properties given by elements of a Boolean sublattice $L_C^\alpha \subset L_Q$. Hence a quantum system is (at a certain time value t) only a carrier of a class of mutually commensurable properties (Mittelstaedt, 1987, pp. 128ff, 1994; Busch *et al.*, n.d.).

On the basis of the general concept of a quantum object as carrier of properties of L_Q one can again specify this concept by considering different classes. Elementary quantum systems are given by *irreducible* unitary representations $S(G)$ of the Galilei group. For elementary objects there are no true representations of the group G_{10}, but only projective ones. These representations are again characterized by one continuous parameter m which can be interpreted as the mass of the quantum object and which characterizes a certain class of objects.

In a similar way one can also use the unsharp properties mentioned above for the definition of a quantum system S_Q. In this case one has to make use of effect-valued measures which fulfill the covariance postulate (C_{QE}). A quantum object can then be defined as a carrier of these unsharp properties, the algebra of which is given by $E(H)$. In contrast to the definition of objects by means of sharp (PV) properties, in case of unsharp (POV) properties an object can possess in principle an arbitrary set of effects $E \in E(H)$ simultaneously. Hence a quantum system can alternatively be defined as carrier of all properties which are subject to the covariance postulate (C_{QE}) and which are unsharp in the sense of POV-measures.

3.4. Individual Quantum Systems

Since quantum objects are carriers of those properties (observables) which fulfill the covariance postulate, for the characterization of individual objects one can use only properties of this kind. Furthermore, due to the general incommensurability of quantum properties, only a Boolean sublattice of the propositional lattice L_Q can simultaneously be applied to the system. This means that either the position observables $\mathbf{q}_i$ or the momentum observables $\mathbf{p}_i$ can be attributed to the system at the same time t, but not both together. Since impenetrability is only known to hold in position space, we try to determine objects by their position observables $\mathbf{q}_k$ which fulfills the covariance conditions with respect to the Euclidean group. Even if uniqueness is given by a contingent impenetrability law at a time t, it must be further guaranteed that the system can be reidentified at a later time t'. This is only possible if the first measurement at time t was a repeatable measurement.

However, according to well-known results (Ozawa, 1984), repeatability implies the discreteness of the measured observable. Since the position observable $E^k(x)$ is continuous, it cannot be measured repeatably and hence it is not possible to reidentify an object by measurements of its position. There are of course well-known procedures to discretize a continuous observable. Any partition (X_i) of the real line $\mathcal{R}$ induces a discrete version $i \rightarrow E^k(x_i)$ of the position observable. However, according to the covariance principle mentioned above, this and other discretization procedures of the position observable $E^k(x)$ are not tenable since they would destroy the covariance with respect to the Euclidean group. It is obvious that the Euclidean covariance must be fulfilled if the position observable $E^k(x)$ shall pertain to the system as one of its properties. In this situation one could try to replace the PV-measure $E^k(x)$ by some convenient POV-measure $E^k_\mu(x)$ which corresponds to an unsharp observable. However, it can be shown again (Busch *et al.*, 1991) that this observable can be measured in a repeatable way only if it is discrete.

A possible way to preserve Euclidean covariance and to reidentify the system at least approximately is to relax the repeatability condition. This can be done in several ways. Here we refer to the almost repeatable measurements, which are also (Busch *et al.*, 1991) called δ-repeatable: For any $\delta > 0$ and any set X we write $X_\delta := \{x \in \mathcal{R}: |x - x'| \leq \delta \text{ for all } x' \in X\}$. A position measurement is said to be δ-repeatable if the probability $p^E(W_S(\varphi, X), X_\delta)$ for obtaining a result of the repeated measurement in X_δ if the result of the previous measurement was in the set X, equals 1. [$W_S(\varphi, X)$ is the final state of the first measurement performed on system S with preparation φ and a result in X.]

If the result of the first measurement is in X, one can alternatively relax the repeatability condition by the requirement that the probability $p^E(W_S(\varphi, X), X)$ for obtaining a result of the repeated measurement in the same set X, i.e., to prepare the system with a value in X, fulfills the relation $p^E(W_S(\varphi, X), X) \geq 1 - \epsilon$ for some ϵ with $0 \leq \epsilon \leq 1$. Measurements of this kind are called ϵ-preparatory. It is obvious that one can further relax the concept of repeatability by combining δ-repeatability and ϵ-preparatory to $(\epsilon - \delta)$-repeatable measurements, for which it holds that $p^E(W_S(\varphi, X), X_\delta) \geq 1 - \epsilon$.

The δ-repeatable measurements do not provide a strict reidentifiability, but only an approximate one. Moreover, only unsharp position observables in the sense of POV-measures can be used for the reidentification. The reasons are that joint measurements of position and momentum are required for making use of a causality law and that the position observable $E^k_\mu(x)$ at the initial time t must be compatible with the position observable $E^k_{\mu'}(x)$ at time

$t' > t$. These two necessary conditions for the reidentification can only be achieved by unsharp position (and momentum) observables, but not by the corresponding PV-measures.

For phase space observables which are needed here, there are neither δ-repeatable nor ϵ-preparatory measurements (Busch *et al.*, n.d.). However, for a given ϵ one can always choose a δ such that this measurement is $(\epsilon - \delta)$-repeatable. For this reason the reidentification must be performed by $(\epsilon - \delta)$-repeatable measurements of the unsharp joint position and momentum observables. We call this unsharp way to reidentify an individual system $(\epsilon - \delta)$-reidentification. It is obvious that by means of this method one cannot denote individuals in the strict sense.

Another, perhaps more promising way to characterize individual systems by means of unsharp position and momentum observables is by "weakly disturbing" measurements Scherer (1994). Let S be an object system with a convenient preparation φ. A unitary joint measurement of an unsharp position and momentum observable can then be described by the operator $U(\lambda, \mu, \mathbf{q}_S, \mathbf{p}_S)$, where $\mathbf{q}_S$, $\mathbf{p}_S$ are the position and momentum operators of the object S, respectively. λ and μ are parameters which determine the strength of the measuring interaction.

Assume that for the special preparation φ a measurement of this kind leads to a final state $W_S(\varphi, \Delta_q \times \Delta_p)$ with a position value in $\Delta_q \in B(\mathfrak{R}_q)$ and a momentum value in $\Delta_p \in B(\mathfrak{R}_p)$. The probability $p(W_S(\varphi, \Delta_q \times \Delta_p), \Delta_q \times \Delta_p)$ for obtaining by a repeated measurement a result in $\Delta_q \times \Delta_p$, if the result of the previous measurement was in $\Delta_q \times \Delta_p$, depends on λ and μ. However, it turns out that λ and μ can be chosen such that $p(W_S, \Delta_q \times \Delta_p) = 1 - \epsilon$, where $\epsilon \geq 0$ is arbitrary small. These state-preserving measurements are "weakly disturbing" measurements. Clearly, measurements of this kind depend on the initial preparation. On the other hand, these "weakly disturbing" measurements can be used for tracing an object system along its trajectory in space-time.

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